Realising potential

Exploring common misunderstandings in GCSE Maths

How to read and approach maths questions and avoid common mistakes

Contents

Help prepare your GCSE students with confidence

Every year in GCSE Maths exams, students often misread, misunderstand or misinterpret questions and don't always do what the question is asking them to do.

This booklet has been designed by our curriculum experts for you to use with your students to explore and highlight some of these common misunderstandings in GCSE Maths assessments.

There are example workings, examiner commentaries, good exam technique and best practice approaches.

Confusing of and off

A frequently used non-calculator question is to ask for a percentage of an amount.

A number of students misread ‘of’ and ‘off’, so instead work out a percentage off a number.

Example question

Work out 20% of 14 000

Example response

10% = 1400

1400x2 = 20%

= 2800

14000 − 2800

= 20% = 11200

Answer 11200

Top tip

A number of students instead work out 20% **off** 14 000. We’d never word the question in this way.

Key considerations

If students show working for 20% off 14 000 they are likely to only lose one mark, but it’s an unnecessary mark to lose.

In the example response, the student knows how to work out 20%, but has finished by taking 20% **off** and can only get one of the two marks.

Thinking we've made a mistake on the paper

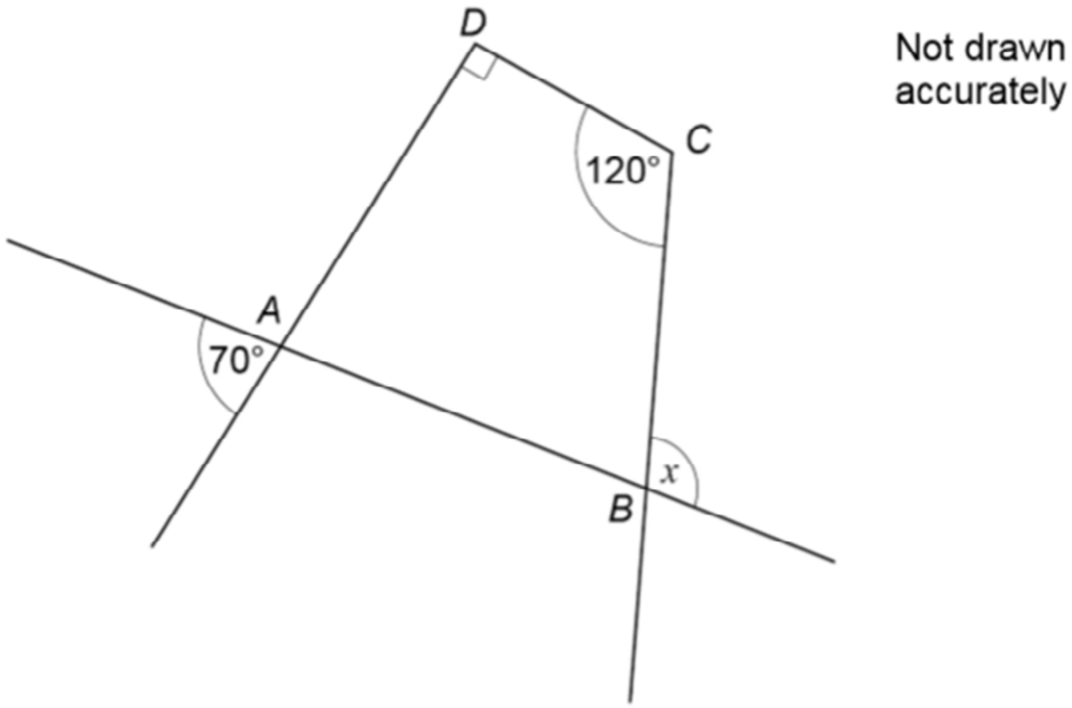
It's common for us to ask students to show that a particular result is true.

We also ask students to find a mistake in someone's work or a diagram. Some students incorrectly attempt to challenge these types of question.

Example question 1

ABCD is a quadrilateral.

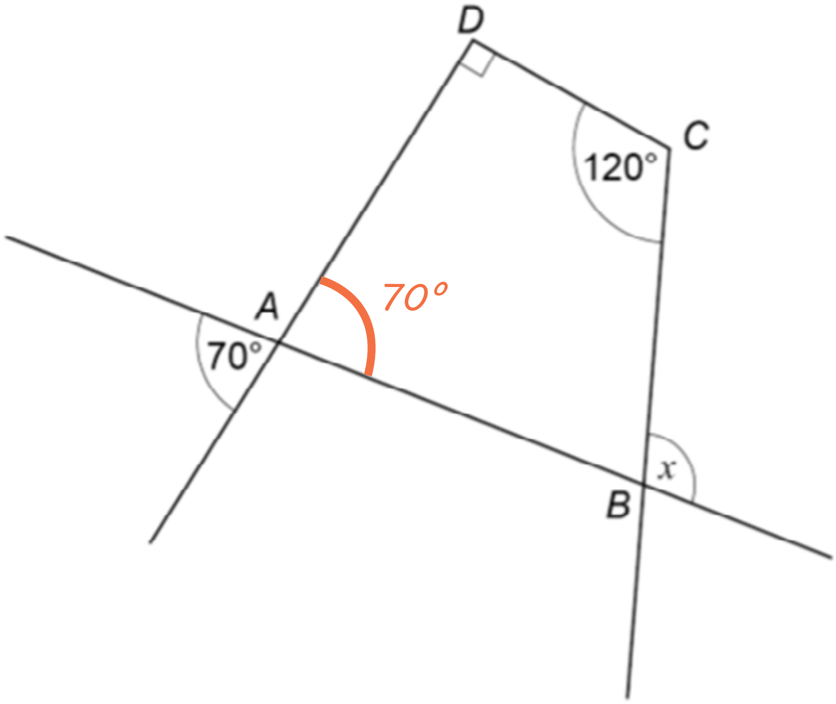
Sides are extended as shown.



Show that $x = 100°$

[3 marks]

Example response 1



x = 100° – not true

x is 70° it is corresponding angle to the 70° at A

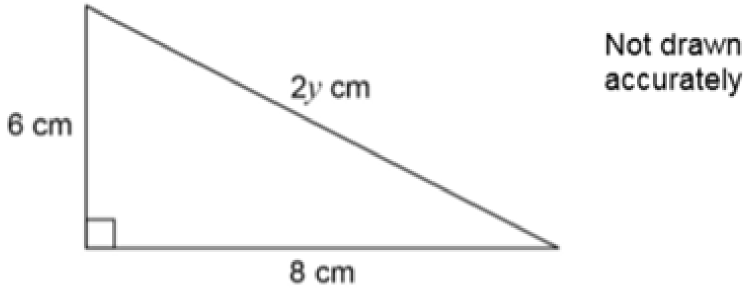
Key considerations

If students don't get the same result as the one we're asking for, they've made a mistake.

The problem with the argument made in the example response is that the x is not corresponding to the 70 as the lines DA and CB are not parallel.

Example question 2

Sami is trying to work out the exact value of y using Pythagoras' theorem.



Here is her working.

$$(2y)^2 = 6^2 + 8^2$$
$$2y^2 = 36 + 64$$
$$2y^2 = 100$$
$$y^2 = 100 \div 2$$
$$y^2 = 50$$
$$y = \sqrt{50}$$

What error has she made in her working?

[1 mark]

Example response 2

No error, Sami has done it right

Top tip

We're not trying to trick students, if we're asking for an error, there's an error to be found.

Key considerations

This response incorrectly states that there's nothing wrong with the work or diagram.

Example question 3

Rulers	Pens
85p each	£3.50 each

Jenny buys 5 rulers and 2 pens.

She works out how much she should pay.

5 x 85p = £4.25

2 x £3.50 = £6.10

Total = £10.35

Jenny's total is wrong. What mistake has she made?

Include the correct total in your answer.

[2 marks]

Example response 3

Mistake made 2 x 3.50 = £7.00 and not £6.10 and 85p x 5 = £6.50 not £4.25 So £7.00 + £6.50 = 13.50

Correct total £ 13.50

~~£4~~

x	80	5
5	40	25

6.50

Key considerations

The wording of the question has 'mistake' as singular. This example response has indicated additional mistakes that aren't actually there. Encourage students to read questions carefully to determine how many mistakes there are.

Misreading the question

It's common for students to misread a number in a question or a question's meaning.

Example question 1

A college has

a total of 105 teachers

19 more female teachers than male teachers.

What proportion of the teachers are female?

[3 marks]

Top tip

If we can follow the student's working we'll award appropriate method marks for working shown, even if the wrong value is used.

Key considerations

This question was quite widely misread as 19 female teachers, not '19 **more** female teachers'. That would have made the question too simple, so we wouldn't have been able to award marks for work following. Again, careful reading of the question is an important message to give students.

Example question 2

8.6	0.27	6.3	0.4

(a) Choose **two** of the cards to make the answer to this calculation a whole number.

Include the answer to the calculation.

☐ + ☐ = ☐

[2 marks]

(b) Choose **two** of the cards to make the answer to this calculation as large as possible.

Include the answer to the calculation.

☐ – ☐ = ☐

[2 marks]

Key considerations

In part (b) of this question, lots of students just used the largest numbers in the calculation and didn't get the largest possible answer. This could have been a misread or a misconception about subtraction.

Different meanings of the word 'estimate'

Students should understand the difference between the two meanings of 'estimate' and when to spot them.

In non-calculator papers we often ask students to estimate the value of a calculation, often prompting to round to the nearest 10 or to 1 significant figure, shown in the first example question below.

We also have to use the word 'estimate' for the mean of a grouped frequency table, as shown in the second example below.

Example question 1

By rounding each number to the nearest 10, estimate the value of $262 \div 19.8$

[2 marks]

Example question 2

Here is some information about 20 trains leaving a station.

Number of minutes late, t	**Number of trains**	**Midpoint**	
$0 \le t < 5$	12		
$5 \le t < 10$	7		
$10 \le t < 15$	1		
$t \ge 15$	0		

Work out an estimate of the mean number of minutes late.

[3 marks]

Example response 1

Number of minutes late, t	Number of trains	Midpoint	
$0 \leqslant t < 5$	12	2.5	30
$5 \leqslant t < 10$	7	7.5	52.5
$10 \leqslant t < 15$	1	12.5	12.5
$t \geqslant 15$	0		

Work out an estimate of the mean number of minutes late.

[3 marks]

$$\frac{30 + 50 + 10}{20} = \frac{90}{20}$$

Answer 4.5 minutes

Top tip

We ask students to 'calculate an estimate' or 'work out an estimate' of the mean value because the exact mean isn't known.

Key considerations

This method requires the use of exact calculations on midpoints and no rounding should be carried out.

It's an estimate because we don't know the exact values of the data within each group, with the midpoints representing them in the best way possible.

Occasionally, we see work done like the first example response above, where rounding is inappropriately carried out.

Example response 2

Number of minutes late, t	Number of trains	Midpoint	FX
$0 \leqslant t < 5$	12	× 2.5	30
$5 \leqslant t < 10$	7	× 7.5	52.5
$10 \leqslant t < 15$	1	× 12.5	12.5
$t \geqslant 15$	0	0	0

Work out an estimate of the mean number of minutes late.

[3 marks]

Mean = add all data ÷
by how many
30 + 52.5 + 12.5 = 95
95 ÷ 20 = 4.75

Answer 5 minutes

Key considerations

This response has also rounded, but only at the very end, so we can see a fully correct answer before any rounding and award full marks.

Unnecessary evaluation of indices

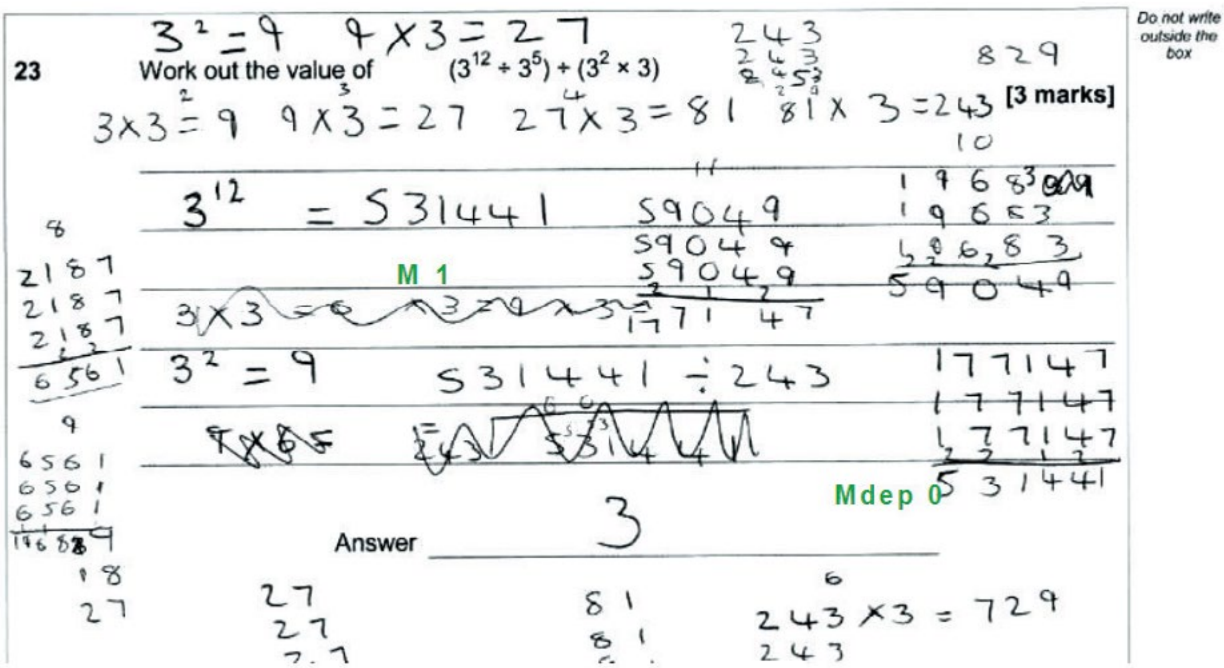
A fairly common style of question on non-calculator papers is to ask for simplification of indices. Some students try, unsuccessfully, to evaluate indices.

Example question

Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

[3 marks]

Example response 1


23 Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$ [3 marks]

Do not write outside the box

$3^2 = 9 \quad 9 \times 3 = 27$

$3 \times 3 = 9 \quad 9 \times 3 = 27 \quad 27 \times 3 = 81 \quad 81 \times 3 = 243$

$3^{12} = 531441$

M 1

$3^2 = 9 \quad 531441 \div 243$

Mdep 0

Answer 3

$243 \times 3 = 729$

Key considerations

Sometimes we ask for the answer as a power but, in many cases we don't. We don't expect students to spend lots of time evaluating; it's not likely to be successful. In this example response, the student tries to work out 3^{12} – this is beyond what we would ask for on a non-calculator paper.

Example response 2

BIDMAS

$(3^{12} \div 3^5) = 3^7$

$(3^2 \times 3) = 3^3$

$3^7 \div 3^3 = 3^4$

3
9 ×3
27 ×3
81

Answer 81 3

Top tip

Remind your students that evaluating powers instead of using rules of indices means lost precious time.

Key considerations

This response shows just how quickly the solution can come once you use the rules of indices.

Simplifying fully or factorising fully

Students won't always notice when they're asked to simplify or factorise fully or don't understand how to get to the correct answer.

When we use the word ‘fully’ in simplifying questions, it’s to make sure students look for the maximum possible amount of simplifying and don't just consider one factor or one change.

Example question 1

Simplify fully $\dfrac{x^2 + 9x + 14}{x^2 - 4}$

....

Example response 1

$((x+2)$

$$\frac{x^2+7x+2x+14}{x^2-4} = \frac{(x+7)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$\frac{x+7}{x-2}$$

Answer $\dfrac{x+7}{x-2}$

Top tip

Even if the word ‘fully’ isn't in the question, students should still go as far as correct mathematics allows.

Key considerations

There will usually be something that cancels from the numerator and the denominator, so students should include thinking about the likelihood of a common factor on each appearing. The first example response shows how this is done.

Example response 2

$x^2 + 9x + 14 = (x+7)(x+2)$

$x^2 - 4 = (x+2)(x-2)$

Answer

Key considerations

This example response has correctly factorised but not spotted the opportunity to cancel, possibly because the identical brackets have not fallen directly in line.

Example question 2

Factorise fully $24y^2 - 20y$

Example response 3

Answer: $y\ (24y - 20)$

Example response 4

Answer: $4(6y^2 - 5y)$

Key considerations

More straightforward factorising questions are often only partly done by students. In the two example responses above, neither has gone to the fully correct answer of $4y(6y-5)$

The number of marks doesn't equal the number of correct values

Students shouldn't assume the number of marks for a question tells you how many answers there are.

Example question 1

Write down **all** the factors of 18

[2 marks]

Top tip

The number of marks allocated to a question is related to the amount of work required to obtain the fully correct answer.

Key considerations

The question's worth two marks, but there are actually six factors of 18: 1, 2, 3, 6, 9, and 18.

Example question 2

Write down **all** the prime numbers between 40 and 50

[2 marks]

Key considerations

The question's worth two marks, but there are three prime numbers between 40 and 50: 41, 43 and 47.

Example question 3

Solve $x^2 = 196$

[2 marks]

Key considerations

Sometimes the marks can be a clue to what's needed.

This example question was from a calculator paper. We wouldn't award 2 marks for square rooting 196 on a calculator to get 14. The second mark is for finding the second solution to this equation, –14

Not taking hints when we give them

There are several ways that we display questions to help save students time and unnecessary work, but they don't always realise.

For example, sometimes we ask students to list all the possible outcomes in a listing situation.

Example question 1

A shop sells ice creams.

Each ice cream has two scoops.

The possible flavours are vanilla (V), strawberry (S), chocolate (C) and mint (M).

The two scoops can be the same flavour or different flavours.

(a) List all the possible options for the two scoops.

[2 marks]

....

Example response 1

Vanilla/Vanilla	~~V+V~~
Vanilla/Strawberry	~~V+S~~
Vanilla/Chocolate	~~V+C~~
Vanilla/Mint	~~V+M~~
Strawberry/Chocolate	~~S+C~~
Strawberry/Strawberry	~~S+S~~
Strawberry/Mint	~~S+M~~
Chocolate/Chocolate	~~C+C~~
Chocolate/Mint	~~C+M~~
Mint/Mint	~~M+M~~

Top tip

We put the letters in brackets after the names to save students time. There's no need to write out the full names.

Key considerations

This example response originally listed just the letters, then decided it might not be accepted and so has lost time listing them out in full.

Example response 2

	V	S	C	M
V	VV	VS	VC	VM
S	SV	SS	SC	SM
C	CV	CS	CC	CM
M	MV	MS	MC	MM

Key considerations

This response shows a nice grid method and use of the initials. No credit was lost for listing flavours in both orders, eg CV and VC.

Example question 2

Date	Description	Credit (£)	Debit (£)	Balance (£)
13/12/2016	Starting balance			212.48
14/12/2016	Council tax		128.39	________
15/12/2016	Salary	856.21		________

Complete the bank statement.

[2 marks]

Top tip

If it's greyed out, it's not needed.

Key considerations

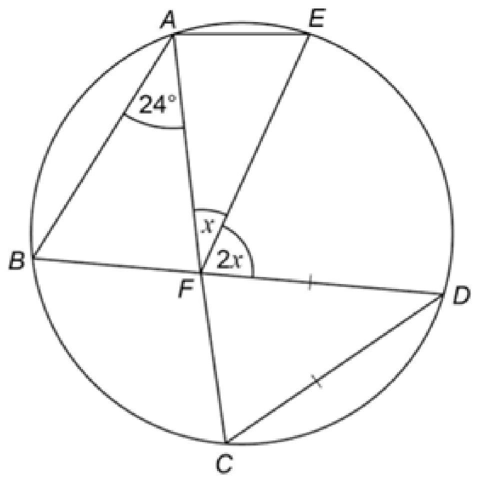
Bank statement questions are quite common in our papers. Students only need to write in the white, blank cells.

Example question 3

A, *B*, *C*, *D* and *E* are points on a circle.

BFD and *AFC* are straight lines.

DC = *DF*



Not drawn accurately

Work out the size of angle x.

You **must** show your working which may be on the diagram.

[4 marks]

Top tip

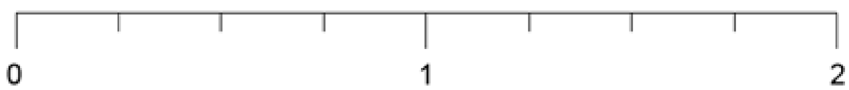
If the question says students can write on the diagram, it's a big hint that it's probably the easiest way to show their working.

Key considerations

In lots of questions involving angles we can give students marks for work they do on the diagram.

Example question 4

Work out the fraction that is halfway between $\frac{1}{2}$ and $1\frac{1}{4}$



[3 marks]

Key considerations

In this example question, we helped by putting a scale on a question asking about half-way points. Using the scale was a good way of getting part marks if students couldn't complete the whole question.

Using ruler and compasses

If we ask students to use a ruler and compasses, then we need to see use of it in their answer. We're able to spot if rulers and compasses haven't been used.

Example question 1

Using ruler and compasses, show the region inside the grid that is

less than 4 cm from *A*

and

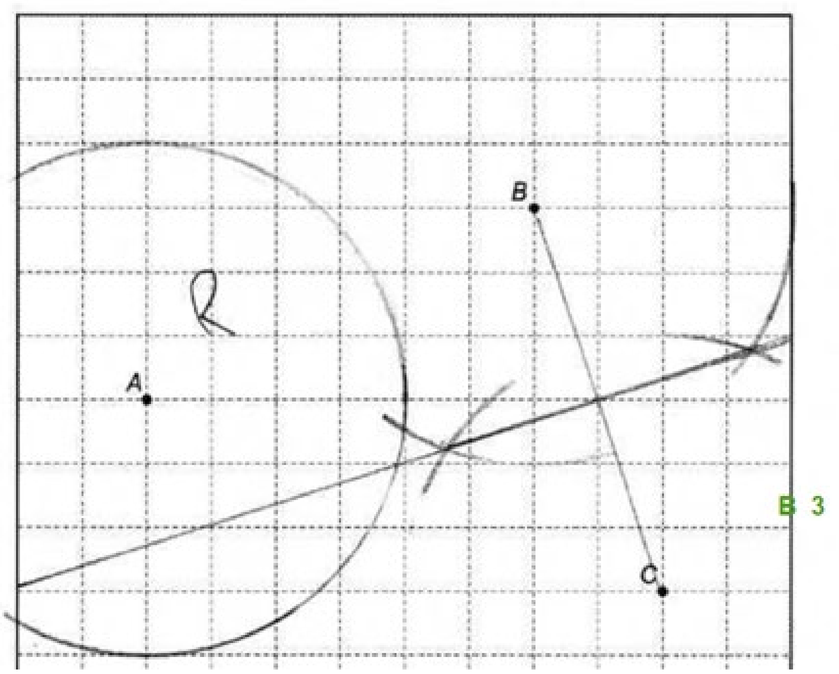
nearer to *B* than to *C*.

Label the region *R*.

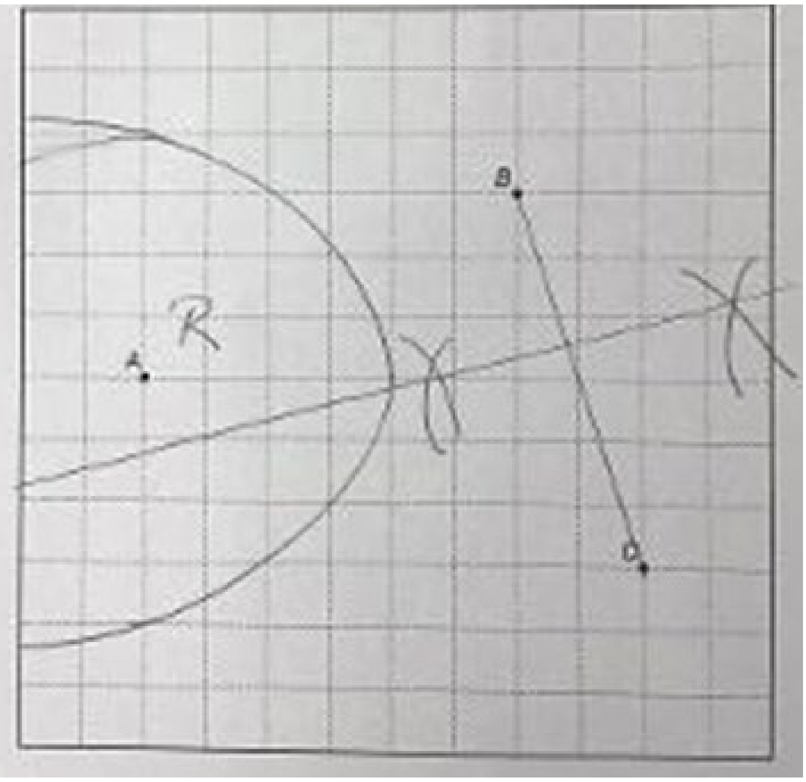
Show all your construction lines.

[3 marks]

Example response 1



Example response 2



Top tip

Students must show the required construction lines and/or arcs to be able to score any marks.

Key considerations

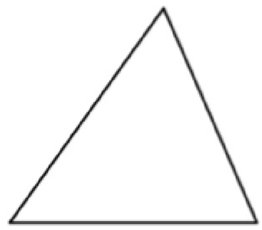
The first example response shows clear use of compasses. However the second response tried to convince us without using the right equipment. We couldn't give any marks for this response.

Assuming things are true that are not

Sometimes students make wrong assumptions about the situation and make costly mistakes as a result.

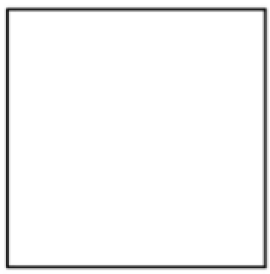
Example question

A triangle has perimeter 32 cm

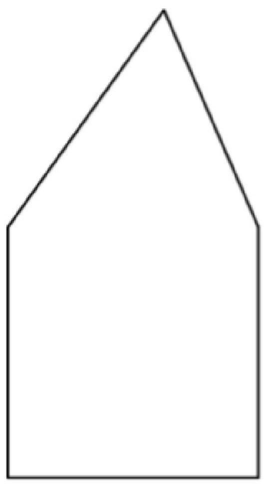


A square has perimeter 40 cm

Not drawn accurately



Two sides of the shapes are put together to make a pentagon.



Not drawn accurately

Work out the perimeter of the pentagon.

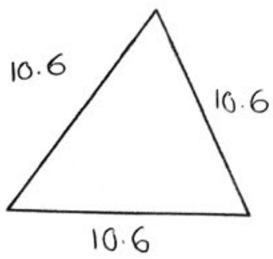
[4 marks]

....

Example response

A triangle has perimeter 32 cm

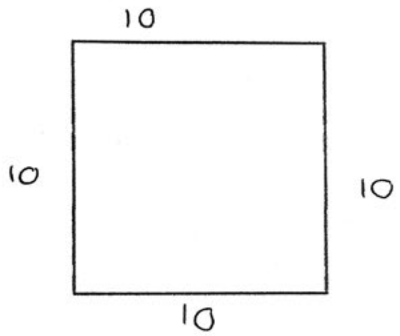
32 ÷ 3
= 10.6



$$3\,\overline{)\,32.{}^{2}0}\quad = 10.6$$

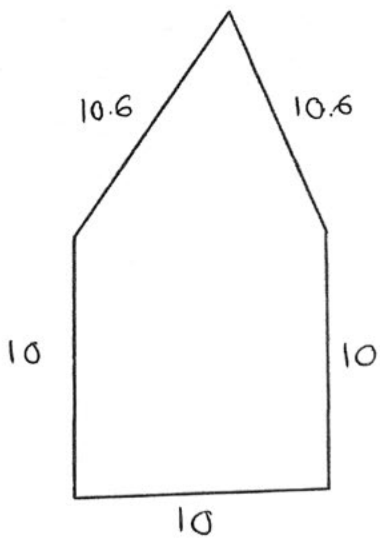
Not drawn accurately

A square has perimeter 40 cm



Two sides of the shapes are put together to make a pentagon.

Not drawn accurately



Key considerations

There's nothing in the question that says the triangle is equilateral but many students made this assumption rather than reasoning that the 'common' side must be 10.

The unusual aspect of this question which may have worried students is that it's impossible but unnecessary to know the length of each side of the triangle.

Realising potential

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November 2021

G02273_48